An Agent-Based Approach to the Cournot Oligopoly Model

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Workshop on
Agent-Based Approaches in Economics
January 24, 2008

Overview

- Agent-based research in micro-economics
- An agent-based approach to the Cournot oligopoly model

Agent-based research in micro-economics

Within the field of micro-economics, there are three subfields in which agent-based approaches are used:

- Evolutionary game theory
- Experimental economics
- Agent-based computational economics

Mainstream micro-economics (1)

- Based on the very strong assumption of fully rational agents
- Takes a deductive approach
- Uses an analytical/mathematical methodology (game theory)

Mainstream micro-economics (2)

Advantages:

- Provides a solid framework for economic analysis
- Provides a lot of insight using simple models
- Reveals cause-effect relationships

Disadvantages:

- Assumptions may be unrealistic
- May not have a good fit with reality
- Only static equilibrium analysis is possible
- Only simple models can de analyzed

Evolutionary game theory (1)

- Relaxes the assumption of fully rational agents
- Uses simple models of boundedly rational agent behavior (e.g., imitation, experimentation, evolution)
- Uses an analytical/mathematical methodology

Evolutionary game theory (2)

Advantages :

- Assumptions may be more realistic
- Provides a lot of insight using simple models
- May explain how equilibria are reached

Disadvantages :

- Assumptions may still be unrealistic
- Only simple models can de analyzed
- Focuses strongly on long-run behavior
- Large variety of models, usually with very different predictions

Experimental economics (1)

- Relaxes the assumption of fully rational agents
- Uses simple models of boundedly rational agent behavior, sometimes psychologically inspired (e.g., experimentation, reinforcement learning, aspiration levels)
- Aims to fit these models to data from laboratory experiments

Experimental economics (2)

Advantages :

- More realistic models by taking into account experimental data
- Incorporates psychological knowledge in models
- Focuses on short-run behavior

Disadvantages :

- Inside the laboratory subjects may behave differently than outside
- Models may not be generally applicable due to the problem of induction
- Models may provide less insight

Agent-based computational economics (1)

- Makes few assumptions
- Allows for complex models of boundedly rational agent behavior
- Sometimes incorporates computer science techniques in models (e.g., genetic algorithms, neural networks)
- Uses a simulation methodology

Agent-based computational economics (2)

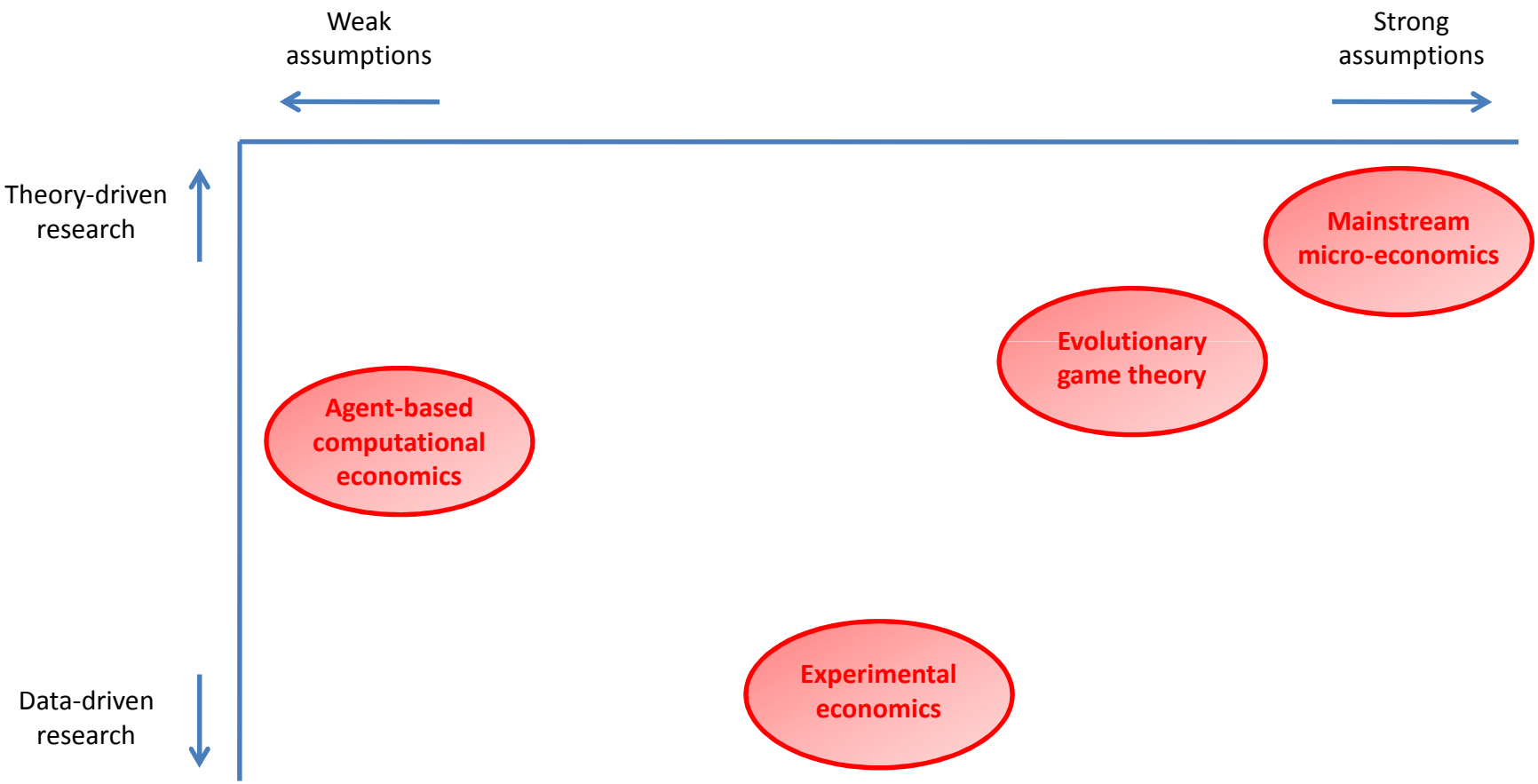
Advantages :

- Needs few assumptions
- Allows for the study of complex models

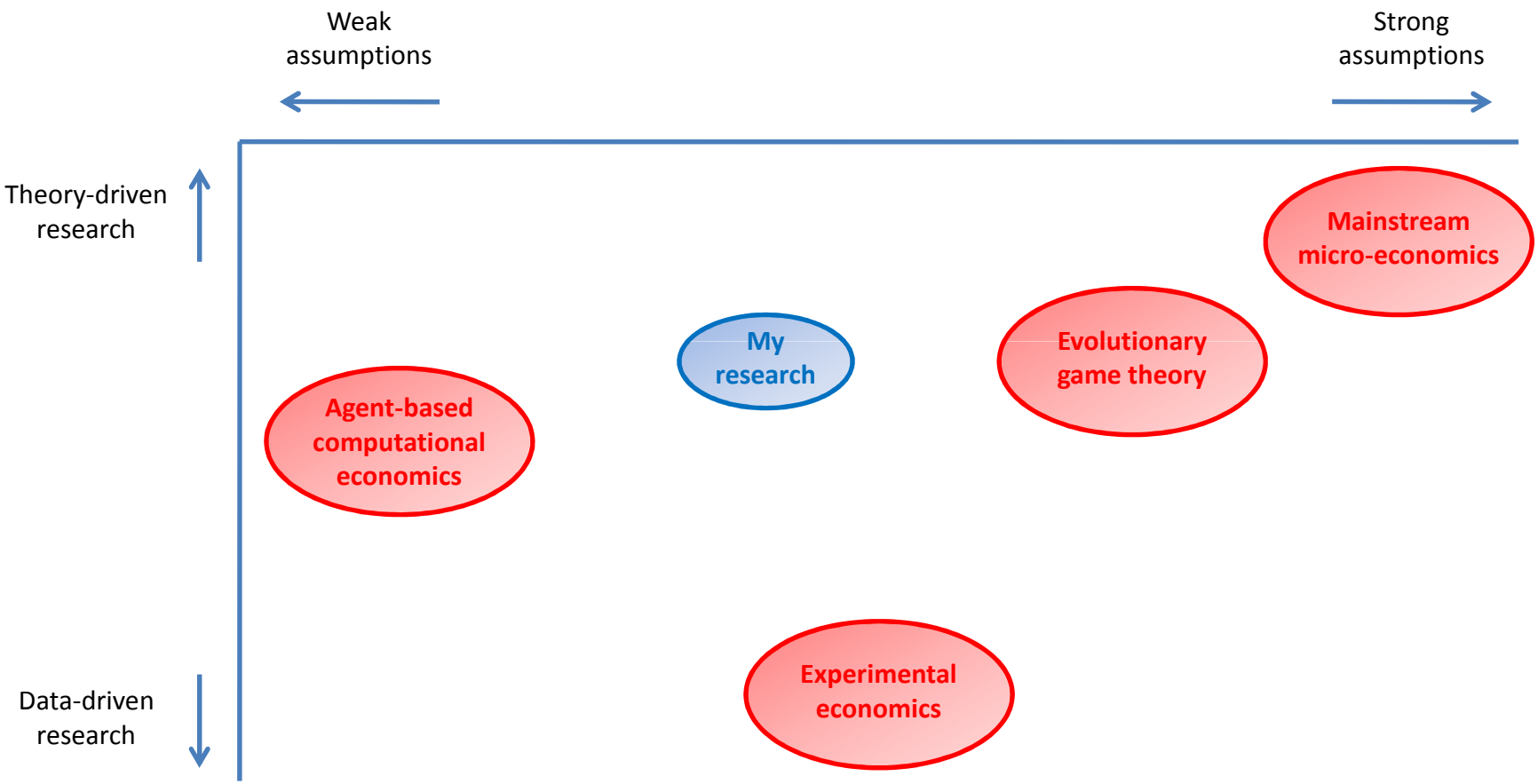
Disadvantages :

- Models are often based on many ad-hoc decisions
- Lots of parameters, sensitivity is often unclear
- Results sometimes do not provide much insight (black box)
- General applicability of the results may be questionable
- No generally accepted methodological standard
- Results are difficult to replicate

Agent-based research in micro-economics



Agent-based research in micro-economics



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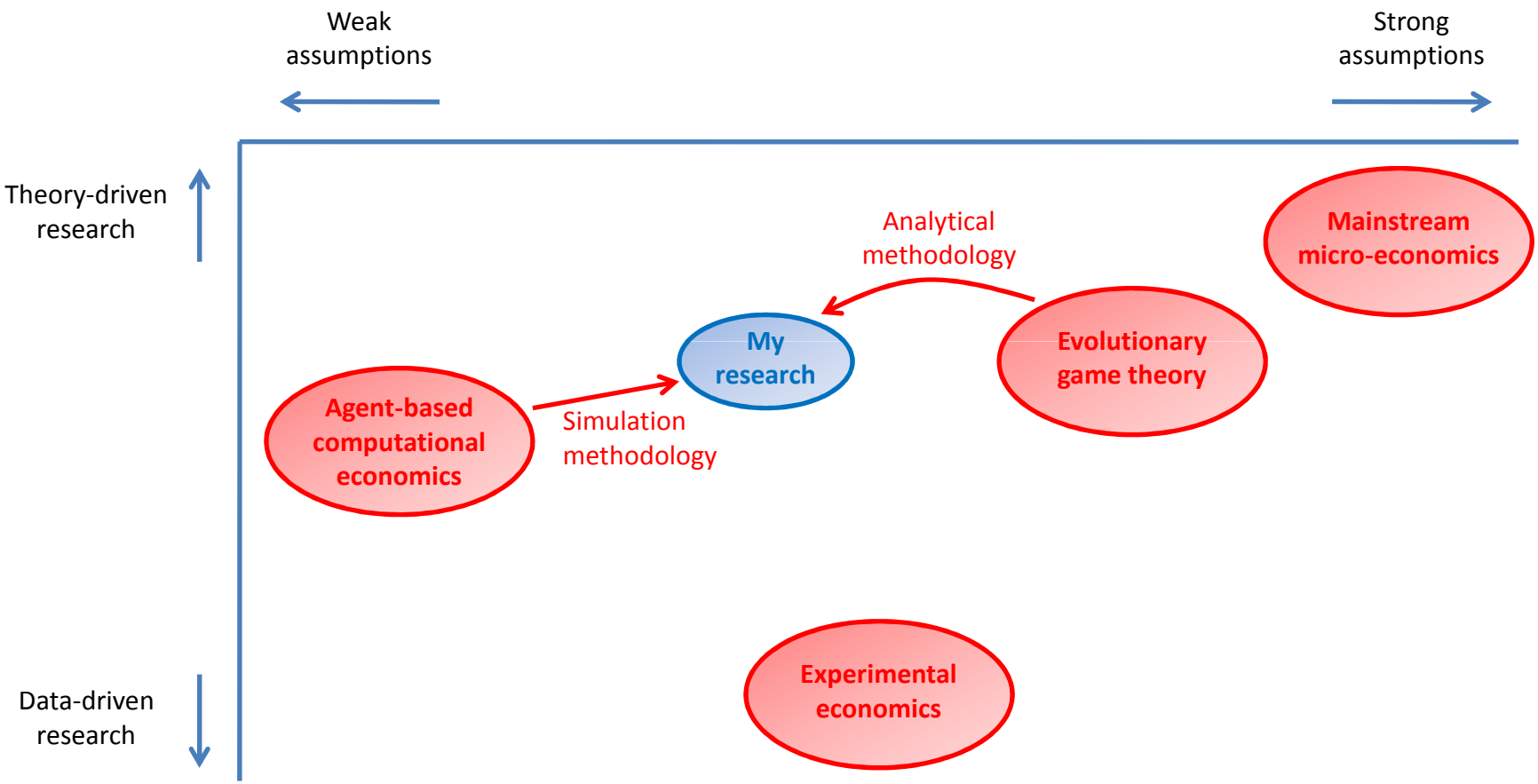
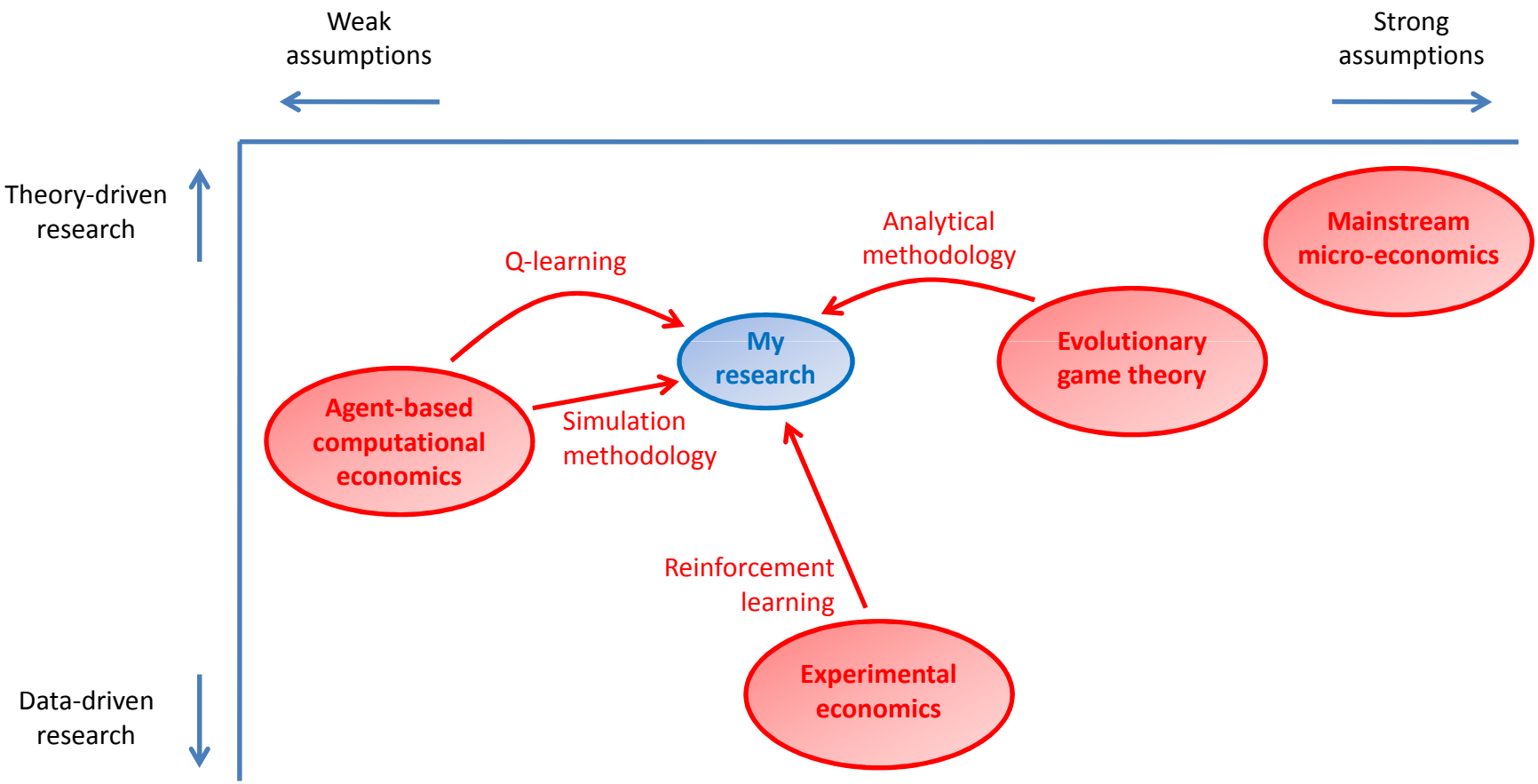


Agent-based research in micro-economics



Main idea and motivation

- Main idea: Use Q-learning as a model of the behavior of firms in a Cournot oligopoly model
- Motivation for Q-learning:
  - In decision problems (no strategic interaction), Q-learning behavior corresponds, in the long run, with the predictions of mainstream economic theory (i.e., utility maximization)
  - Q-learning belongs to the family of reinforcement learning models studied in experimental economics. Hence, at first sight Q-learning appears to be a reasonable model of agent behavior

Methodology

1. First implement a computer simulation in order to
   - quickly get some preliminary results;
   - check whether the results make sense at first sight;
   - check whether the results are interesting and deserve further study
2. Then try to obtain analytical results in order to
   - check the correctness of the simulation results;
   - get more insight into the robustness of the simulation results;
   - get more insight into the underlying cause-effect relationships
3. If necessary, perform additional simulations in order to
   - clarify questions raised by the analytical results;
   - make conjectures how the analytical results generalize

Cournot oligopoly model (1)

- There are two firms (duopoly model)
- Inverse demand function:

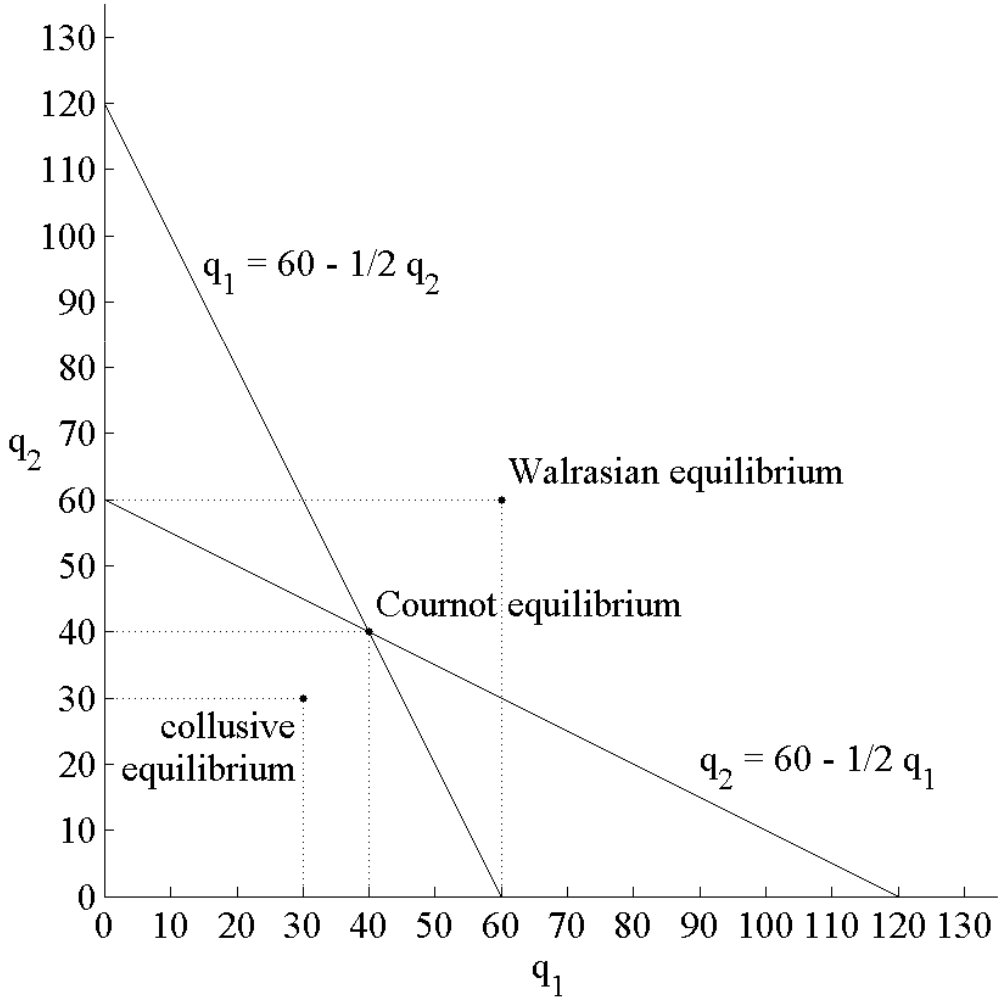
  $p = 150 - \Sigma_i q_i$

  p price
  q_i quantity produced by firm i
- Cost function:

  $c_i = 30q_i$

  c_i total cost of firm i

Cournot oligopoly model (2)



Q-learning

- Action choice rule:

$$\Pr(a) = \frac{\exp(Q_t(a)/\beta)}{\sum_{a'} \exp(Q_t(a')/\beta)}$$

a	a production level
$Q_t(a)$	expected profit of producing a units in period t
β	experimentation tendency parameter

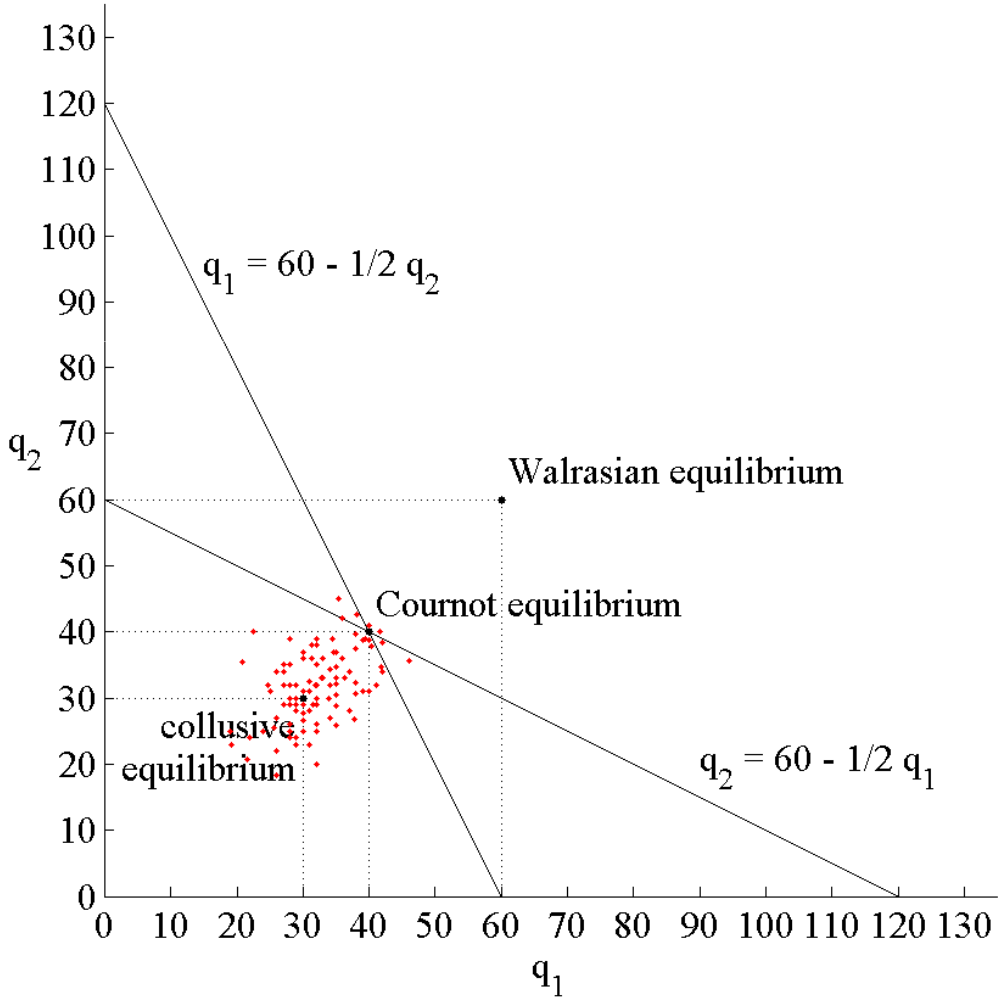
- Update rule:

$Q_{t+1}(a) = (1 - \alpha)Q_t(a) + \alpha\pi_t$ if $a = a_t$

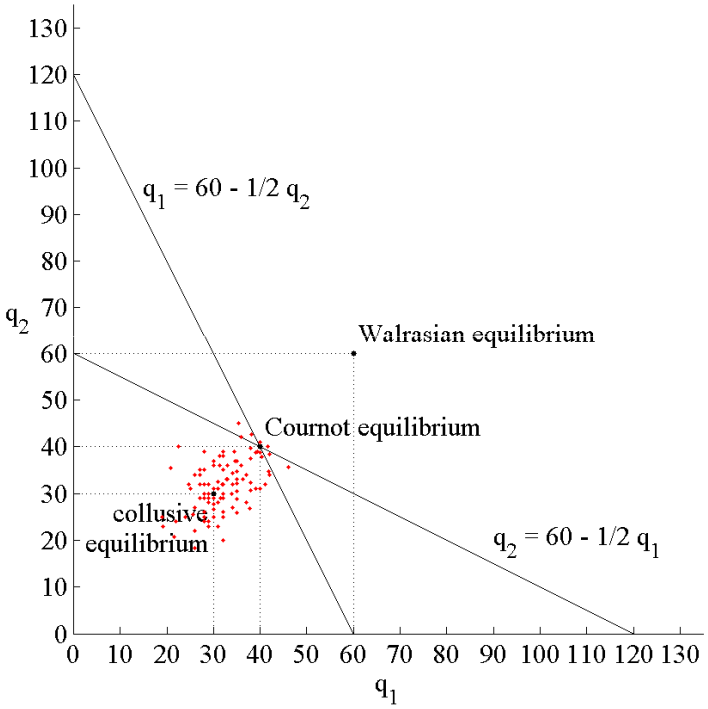
$Q_{t+1}(a) = Q_t(a)$ otherwise

a_t	firm's production level in period t
π_t	firm's profit in period t
α	learning rate parameter

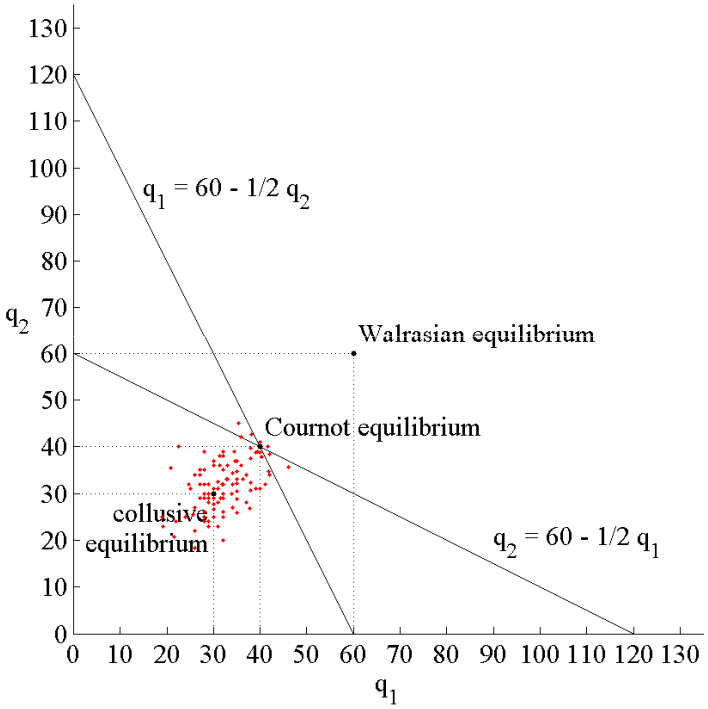
Simulation results



Mathematical analysis (1)



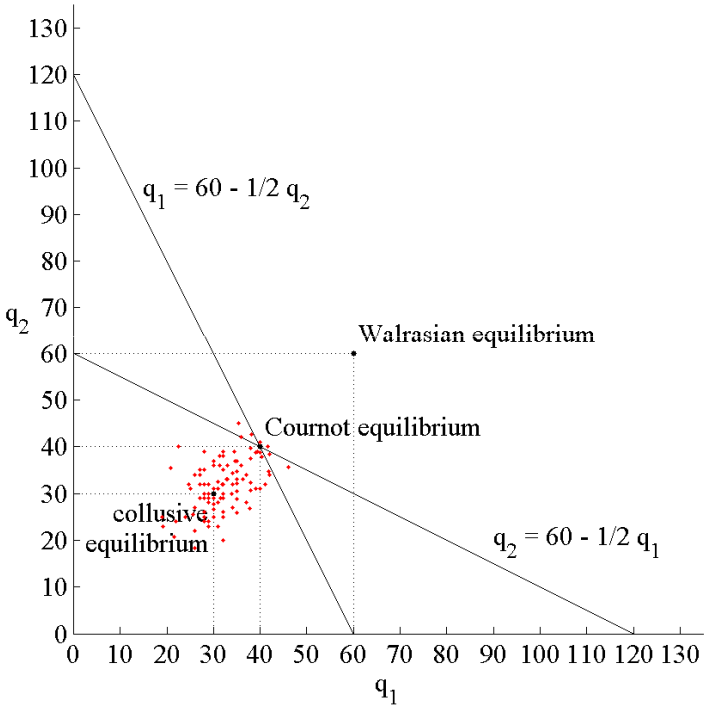
Mathematical analysis (1)



Inverse demand function:	$p = 150 - \Sigma_i\, q_i$
Cost function:	$c_i = 30 q_i$

	$q_2 = 35$	$q_2 = 40$
$q_1 = 35$	(1750, 1750)	(1575, 1800)
$q_1 = 40$	(1800, 1575)	(1600, 1600)

Mathematical analysis (1)



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$q_1 = 40$	(1800, 1575)	(1600, 1600)

Inverse demand function:	$p = u - v(\Sigma_i\, q_i)$
Cost function:	$c_i = wq_i$

	$q_2 = q_C$	$q_2 = q_N$
$q_1 = q_C$	(π_{CC}, π_{CC})	(π_{CN}, π_{NC})
$q_1 = q_N$	(π_{NC}, π_{CN})	(π_{NN}, π_{NN})

Mathematical analysis (2)

Inverse demand function:	$p = u - v(\Sigma_i\, q_i)$
Cost function:	$c_i = wq_i$

	$q_2 = q_C$	$q_2 = q_N$
$q_1 = q_C$	(π_{CC}, π_{CC})	(π_{CN}, π_{NC})
$q_1 = q_N$	(π_{NC}, π_{CN})	(π_{NN}, π_{NN})

Theorem Assume that

$$(u-w)/4v < q_C < q_N = (u-w)/3v$$

$$\frac{\pi_{NN} - \pi_{CN}}{\pi_{CC} - \pi_{CN}} < \alpha < 1$$

Then, in the limit as the experimentation tendency β approaches zero and the time *t* approaches infinity, the probability that both firms produce q_C equals one.

More simulation results

- The mathematical analysis suggests that collusion depends crucially on the action choice rule:

$$\Pr(a) = \frac{\exp(Q_t(a)/\beta)}{\sum_{a'} \exp(Q_t(a')/\beta)}$$

  Simulation results indeed indicate that other action choice rules do not lead to collusion
- The mathematical analysis suggests that collusion also occurs with more than two firms. Simulation results seem to confirm this

Conclusions

- Simulation shows that Q-learning behavior in a Cournot model can lead to collusion
- The emergence of collusion has been formally proven under certain assumptions
- The proof does not depend on the parameters of the demand function and the cost function
- Simulation seems to indicate that some assumptions underlying the analytical results can be relaxed, and that collusion depends crucially on the action choice rule of Q-learning
- *Combining computer simulation and mathematical analysis can be very insightful*